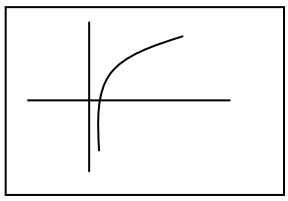
Table of Contents for Mathematical Methods (CAS)

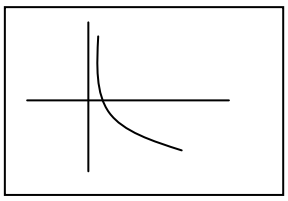
1.2 Graphs of log functions

$y = A\log_a k(x-b)+c$

* a > 1



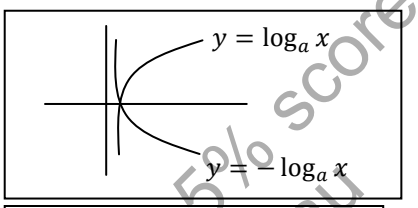
* 0 < a < 1



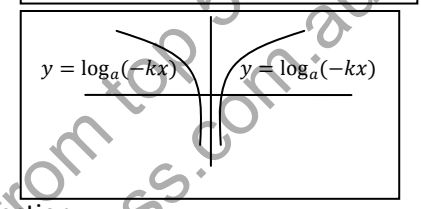
* A: dilating by a factor "A" units parallel to the y-axis (from the x-axis)

* k: dilating by a factor of 1/k parallel to the x-axis (from the y-axis)

* A < 0: reflected in the x-axis



* k < 0: reflected in the y-axis



* b > 0: translated positive b units in the x-direction

* b < 0: translated negative b units in the x-direction

* c > 0: translated c units up in the y-direction

* c < 0: translated c units down in the y-direction

4.2 Product of Function

- product means when two functions multiple together.
- sketching each function first, then multiple the key points together.
- key points include intercepts, turning points etc.
- Asymptotes of any functions are as well asymptotes of the product function.
- The product of two functions will equal zero if one of the functions equal zero at the point.
- If either function equal 1 then the product of the two functions will equal the value of the other function.
- If there are any asymptotes the product of the functions will not be defined at that point.

4.3 Composite Functions

* composite function can be thought of as the function of a function

1. Notation
   - $f \,.\, g(x) = F°g(x) = F[g(x)]$
   - $dom\ f[g(x)] = dom\ g(x)$
   - $f[g(x)]$ is defined if range $g(x) \subseteq$ domain $f(x)$

Example 1: $f(x) = \sqrt{x^2 + 1}$
- domain: R

$g(x) = x^2$
- domain: R
- range $y \geq 0$

* range $g(x) \subseteq$ domain $f(x)$
$(y \geq 0 \quad \subseteq R)$
$f[g(x)] = \sqrt{(x^2)^2 + 1} = \sqrt{x^4 + 1}$

* range $f(x) \subseteq$ domain $g(x)$
$(y \geq 1 \quad \subseteq R)$
$g[f(x)] = (\sqrt{x^2 + 1}\,)^2 = x^2 + 1$

Example 2: $f(x) = x^2 - 4$
- domain: R
- range: $y \geq -4$

$g(x) = 2\sqrt{x}$
- domain: $x \geq 0$
- range: $y \geq 0$

* $\rightarrow f[g(x)]\ exists\ because\ range\ g(x) \subseteq domain\ f(x)$
$\rightarrow \quad f[g(x)] = (2\sqrt{x})^2 - 4 = 4|x| - 4 = 4(|x| - 1)$

* $g[f(x)] does\ NOT\ exist$

8.4 The binomial Probability distribution and its graph

$n\ C_x P^x q^{n-x} = P_r(X = x)$

n= trials , identical

X: even numbers
x: the no. of possible values
P: probability of success
q: probability of failure
p+q=1

Pascal triangle

0
1 1
2 1 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

^6_2C ^5_6C

keyboard,
$^{50}_{30}C$: maths
ncr

(Q 3-438) Example: X= even number n=50
$P_r(X = x) = 0.5$
$P_r(X = 30)$

a) At least 30 even no are rolled

$P_r(X = 30) = {}^{50}_{30}C \times 0.5^{30} \times 0.5^{20}$
$= 0.042$

b) At least 1 six is rolled $P_r(x \geq 1)$

$1 - P_r(x = 0)$

$n = 50, x = 0$
$x: rolling\ a\ 6$
$P = {}^1/_6$

$\rightarrow P_r(x \geq 1) = 1 - {}^{50}_{0}C \times \frac{1}{6}^0 \times \frac{5}{6}^{50}$

* $X \sim B_i$ (20 , 0.25) ← either success or failure
  means X follows a binomial distribution
  number trials : n=20
  pro. success P=0.25
  → $P_r(x = 5) = \binom{20}{5} 0.25^5 \times 0.75^{15} = 0.2023$

D_o in CAS

* Statistic →CAL→ $Distribution \rightarrow Binomial\ PD$
  $\rightarrow enter\ x\ , numtrial\ (n), pos(p)$
* Mode: highest $P_r(X = x)$
  the x value that has the highest value
* Median: $P_r(X < a) = 0.5$

* $E(x) = np$
  $Var\ (x) = n\ .p\ .q = np(1-p)$
  $\sigma = \sqrt{npq} = \sqrt{np(1-p)}$

Example: a fair die is tossed 100 times. What range are you confident 95% that the no. 3 is obtained.

- X: rolling a 3
- n=100
- pro success p=1/6
- $E(x) = n \times p = \frac{100}{6}$
- $Var\ (x) = np(1-p) = \frac{100}{6}\left(1 - \frac{1}{6}\right) = \frac{100}{6} \times \frac{5}{6} = \frac{125}{9}$