

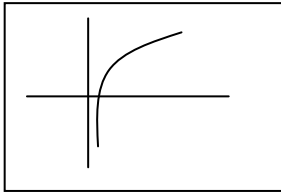
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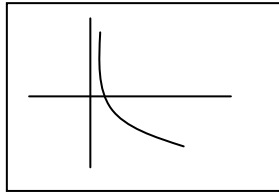
### 1.2 Graphs of log functions

$$y = A \log_a k(x - b) + c$$

\*  $a > 1$



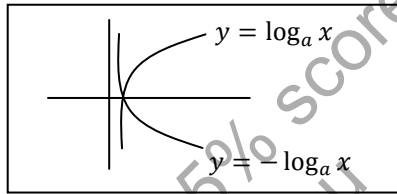
\*  $0 < a < 1$



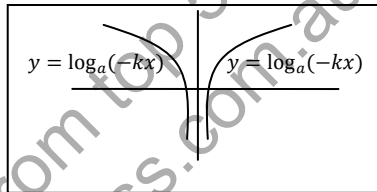
\* A: dilating by a factor "A" units parallel to the y-axis (from the x-axis)

\* k: dilating by a factor of 1/k parallel to the x-axis (from the y-axis)

\*  $A < 0$ : reflected in the x-axis



\*  $k < 0$ : reflected in the y-axis



\*  $b > 0$ : translated positive b units in the x-direction

\*  $b < 0$ : translated negative b units in the x-direction

\*  $c > 0$ : translated c units up in the y-direction

\*  $c < 0$ : translated c units down in the y-direction

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## 4.2 Product of Function

- product means when two functions multiple together.
- sketching each function first, then multiple the key points together.
- key points include intercepts, turning points etc.
- Asymptotes of any functions are as well asymptotes of the product function.
- The product of two functions will equal zero if one of the functions equal zero at the point.
- If either function equal 1 then the product of the two functions will equal the value of the other function.
- If there are any asymptotes the product of the functions will not be defined at that point.

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### 4.3 Composite Functions

\* composite function can be thought of as the function of a function

#### 1. Notation

- $f \circ g(x) = F \circ g(x) = F[g(x)]$
- $\text{dom } f[g(x)] = \text{dom } g(x)$
- $f[g(x)]$  is defined if  $\text{range } g(x) \subseteq \text{domain } f(x)$

Example 1:  $f(x) = \sqrt{x^2 + 1}$   
- domain:  $\mathbb{R}$

$g(x) = x^2$   
- domain:  $\mathbb{R}$   
- range:  $y \geq 0$

\*  $\text{range } g(x) \subseteq \text{domain } f(x)$   
( $y \geq 0 \subseteq \mathbb{R}$ )  
 $f[g(x)] = \sqrt{(x^2)^2 + 1} = \sqrt{x^4 + 1}$

\*  $\text{range } f(x) \subseteq \text{domain } g(x)$   
( $y \geq 1 \subseteq \mathbb{R}$ )  
 $g[f(x)] = (\sqrt{x^2 + 1})^2 = x^2 + 1$

Example 2:  $f(x) = x^2 - 4$   
- domain:  $\mathbb{R}$   
- range:  $y \geq -4$

$g(x) = 2\sqrt{x}$   
- domain:  $x \geq 0$   
- range:  $y \geq 0$

\*  $\rightarrow f[g(x)]$  exists because  $\text{range } g(x) \subseteq \text{domain } f(x)$   
 $\rightarrow f[g(x)] = (2\sqrt{x})^2 - 4 = 4|x| - 4 = 4(|x| - 1)$

\*  $g[f(x)]$  does NOT exist

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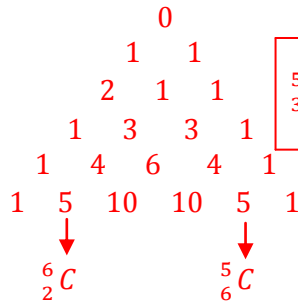
### 8.4 The binomial Probability distribution and its graph

$${}^n C_x P^x q^{n-x} = P_r(X = x)$$

n= trials , identical

X: even numbers  
 x: the no. of possible values  
 P: probability of success  
 q: probability of failure  
 p+q=1

Pascal triangle



(Q 3-438) Example: X= even number n=50  
 $P_r(X = x) = 0.5$   
 $P_r(X = 30)$

keyboard,  
 ${}^{50} C_{30}$ : maths  
 ncr

- a) At least 30 even no are rolled  
 $P_r(X = 30) = {}^{50} C_{30} \times 0.5^{30} \times 0.5^{20}$   
 $= 0.042$
- b) At least 1 six is rolled  $P_r(x \geq 1)$

$n = 50, x = 0$   
 x: rolling a 6  
 $P = 1/6$

$$\rightarrow P_r(x \geq 1) = 1 - {}^{50} C_0 \times \frac{1}{6}^0 \times \frac{5}{6}^{50}$$

$$1 - P_r(x = 0)$$

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- \*  $X \sim B_i$  (20, 0.25)  $\rightarrow$  either success or failure  
 means X follows a binomial distribution  
 number trials :  $n=20$   
 pro. success  $P=0.25$   
 $\rightarrow P_r(x = 5) = \binom{20}{5} 0.25^5 \times 0.75^{15} = 0.2023$

**D<sub>o</sub> in CAS**

- \* Statistic  $\rightarrow$  CAL  $\rightarrow$  Distribution  $\rightarrow$  Binomial PD  
 $\rightarrow$  enter  $x$ , numtrial ( $n$ ), pos( $p$ )
- \* Mode: highest  $P_r(X = x)$   
 the  $x$  value that has the highest value
- \* Median:  $P_r(X < a) = 0.5$

- \*  $E(x) = np$   
 $Var(x) = n \cdot p \cdot q = np(1 - p)$   
 $\sigma = \sqrt{npq} = \sqrt{np(1 - p)}$

Example: a fair die is tossed 100 times. What range are you confident 95% that the no. 3 is obtained.

- X: rolling a 3
- $n=100$
- pro success  $p=1/6$
- $E(x) = n \times p = \frac{100}{6}$

$$- Var(x) = np(1 - p) = \frac{100}{6} \left(1 - \frac{1}{6}\right) = \frac{100}{6} \times \frac{5}{6} = \frac{125}{9}$$

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