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References to an activity, exercise or other type of question are from:

Coffey D, McLaverty J, Anastasio R, Matherson A. (2009). Specialist mathematics Enhanced. Australia: Pearson Heinemann.

What's found in here are our own solutions along with notes on specialist mathematics.

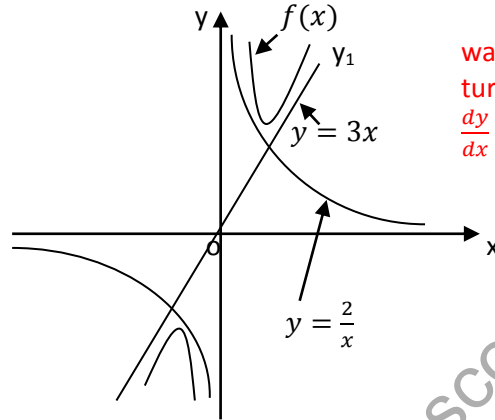
3. Coordinate geometry and graphs

3.1 Addition of Ordinates

- $f(x) = ax + \frac{b}{x}$
 $\Rightarrow f(x) = y_1 + y_2$

- sketch y_1 and y_2 separately then add them together to produce $f(x)$

Example 1: $f(x) = 3x + \frac{2}{x}$

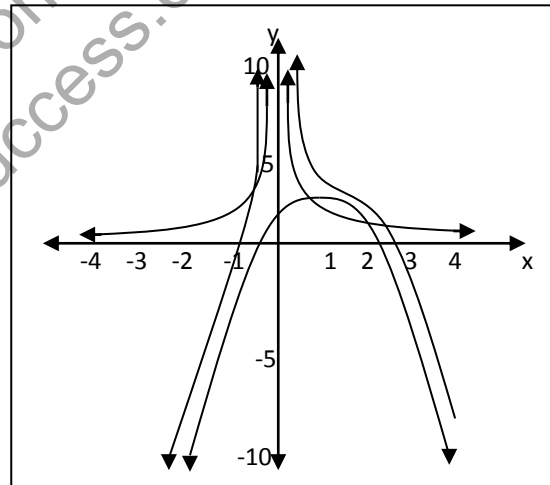
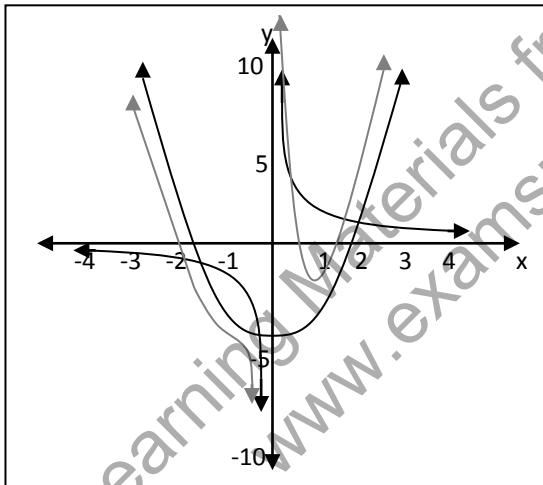


want to find the turning point, find $\frac{dy}{dx}$ of $f(x)$

Example 2: $y = \frac{x^2 - x + 2}{x} = \underbrace{x - 1}_{y_1} + \frac{2}{x}_{y_2}$

$$y = \frac{2}{x} + x^2 - 4$$

$$y = \frac{2}{x^2} - (x - 1)^2 + 2$$



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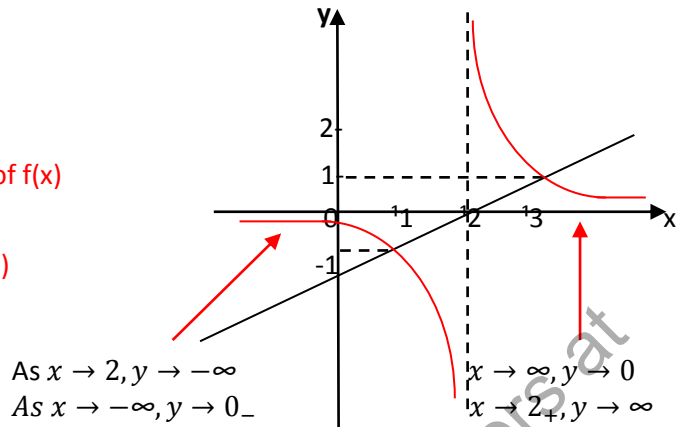
3.2 Reciprocating Ordinates

1. Reciprocating linear functions

Example: $f(x) = x - 2$
 $y = \frac{1}{f(x)} = \frac{1}{x-2}$

Notice: asymptotes at x-intercepts of $f(x)$

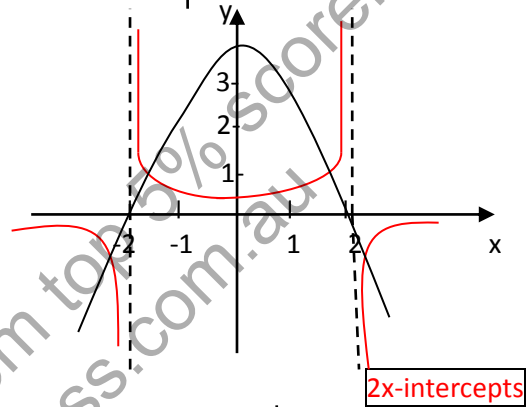
$f(x) = \frac{1}{f(x)}$ at $y = \pm 1$
 (points of intersection at $y = \pm 1$)



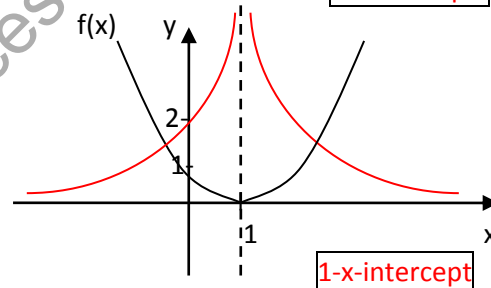
2. Reciprocating quadratic functions

Example 1: $f(x) = 4 - x^2$
 $y = \frac{1}{f(x)} = \frac{1}{4-x^2}$

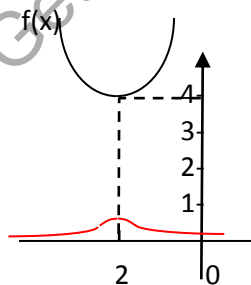
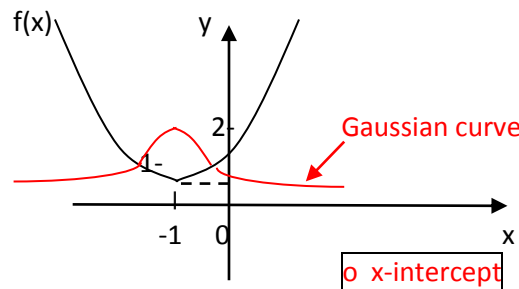
- * Even function has Oy- as the symmetrical axis.
 $\Rightarrow g(-x) = g(x)$
- * Odd function such as $y = x$,
 $y = x^3 \Rightarrow g(-x) = -g(x)$



Example 2: $f(x) = \frac{1}{2}(x-1)^2$
 $y = \frac{1}{f(x)} = \frac{2}{(x-1)^2}$



Example 3: $f(x) = (x+1)^2 + \frac{1}{2}$
 $y = \frac{1}{f(x)} = \frac{1}{(x+1)^2 + \frac{1}{2}}$



Example (4) $\int x(3x^2 - 11)^4 dx$
 $\Rightarrow \frac{1}{6} \int 6x(3x^2 - 11)^4 dx = \frac{1}{6 \times 5} (3x^2 - 11)^5 + c = \frac{1}{30} (3x^2 - 11)^5 + c \quad c \in \mathbb{R}$

Example (5) $\int (15x^2 - 7)(5x^3 - 7x)^7 dx = \frac{1}{8} (5x^3 - 7x)^8 + c \quad c \in \mathbb{R}$
 exactly the differentiate inside

Example (6) $\int \cos x \cdot e^{7 \sin x} dx = \frac{1}{7} \int 7 \cos x \cdot e^{7 \sin x} dx$
 $= \frac{1}{7} e^{7 \sin x} + c \quad (c \in \mathbb{R})$

* $\int \frac{1}{x} dx = \log_e |x| + c, \quad c \in \mathbb{R}$

(top is the derivative of bottom)

Example (7) $\int \frac{2}{2x-5} dx = \log_e |2x-5| + c, \quad c \in \mathbb{R}$

Example (8) $\int \frac{x-2}{x^2-4x} dx = \frac{1}{2} \int \frac{2(x-2)}{x^2-4x} dx = \frac{1}{2} \log_e |x^2 - 4x| + c \quad (c \in \mathbb{R})$

Important notice:

- * The expression $\int f(x) dx$ always needs "+c" in its answer.
- * If the question ask to find " an antiderivative" then please have an extra step at the end to state the final answer without "+c".

Look at example 4 if not clear.

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5.2 Antidifferentiation using change of variable

Substitution method

Example 1. $\int 5x(x^2 + 3)^6 dx = I$, say

$$\begin{aligned} \text{let } u &= x^2 + 3 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \\ \Rightarrow I &= \int 5x \cdot u^6 \cdot \frac{du}{2x} = \int \frac{5}{2} u^6 du = \frac{5}{2} \int u^6 du \\ &= \frac{5}{2} \times \frac{1}{7} u^7 + c = \frac{5}{14} u^7 + c = \boxed{\frac{5}{14} (x^2 + 3)^7 + c \quad c \in \mathbb{R}} \end{aligned}$$

Example 2. $\int 3x\sqrt{x^2 - 1} dx = I$, say

$$\begin{aligned} \text{let } u &= x^2 - 1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \\ \therefore I &= \int 3x \cdot u^{1/2} \cdot \frac{du}{2x} = \frac{3}{2} \int u^{1/2} du \\ &= \frac{3}{2} \times \frac{2}{3} u^{3/2} + c = u^{3/2} + c = \boxed{(x^2 - 1)^{3/2} + c, \quad (c \in \mathbb{R})} \end{aligned}$$

The answer should reflect the style of the question

$$\therefore \text{The final answer} = (x^2 - 1)\sqrt{x^2 - 1} + c$$

Example 3. $\int (3x - 2)\sqrt[3]{3x^2 - 4x} dx = I$, say

$$\begin{aligned} \text{let } u &= 3x^2 - 4x \Rightarrow \frac{du}{dx} = 6x - 4 \Rightarrow dx = \frac{1}{6x - 4} du \\ \therefore I &= \int (3x - 2) \cdot u^{1/3} \cdot \frac{1}{6x - 4} du \\ &= \frac{1}{2} \int u^{1/3} du = \frac{1}{2} \times \frac{3}{4} u^{4/3} + c = \frac{3}{8} u^{4/3} + c \quad (c \in \mathbb{R}) \\ &= \boxed{\frac{3}{8} (3x^2 - 4x)\sqrt[3]{3x^2 - 4x} + c} \end{aligned}$$

Example 4. $\int \frac{5x}{\sqrt{7x^2 + 3}} dx = I$, say

$$\text{let } u = 7x^2 + 3 \Rightarrow \frac{du}{dx} = 14x \Rightarrow dx = \frac{1}{14x} du$$

$$\begin{aligned} \therefore I &= \int \frac{5x}{\sqrt{u}} \cdot \frac{1}{14x} du = \frac{5}{14} \int u^{-1/2} du \\ &= \frac{5}{14} \times 2 u^{1/2} + c = \boxed{\frac{5}{7} \cdot \sqrt{7x^2 + 3} + c \quad c \in \mathbb{R}} \end{aligned}$$

Example 5 $\int 3 \cos x \cdot \sin^2 x dx = I$, say

$$\begin{aligned} \text{let } u &= \sin x \Rightarrow \frac{du}{dx} = \cos x \Rightarrow dx = \frac{1}{\cos x} du \\ \therefore I &= \int 3 \cos x \cdot u^2 \cdot \frac{1}{\cos x} du = 3 \int u^2 du \\ &= \frac{3}{3} u^3 + c = \boxed{\frac{1}{2} \sin^6 x + c \quad (c \in \mathbb{R})} \end{aligned}$$

Example 6 $\int 5 \sin x e^{3 \cos x} dx = I$, say

$$\begin{aligned} \text{let } u &= 3 \cos x \Rightarrow \frac{du}{dx} = -3 \sin x \Rightarrow dx = \frac{-1}{3 \sin x} du \\ \therefore I &= \int 5 \sin x \cdot e^u \cdot \frac{-1}{3 \sin x} du \\ &= \frac{-5}{3} \int e^u du = \frac{-5}{3} e^u + c \\ &= \frac{-5}{3} e^{3 \cos x} + c, \quad c \in \mathbb{R} \end{aligned}$$

Example 7 $\int \frac{-3 \cos x}{\sqrt{4-2 \sin x}} dx = I$, say

$$\begin{aligned} \text{let } u &= 4 - 2 \sin x \Rightarrow \frac{du}{dx} = -2 \cos x \Rightarrow dx = \frac{-du}{-2 \cos x} \\ \therefore I &= \int -3 \cos x \cdot \frac{1}{u^{1/2}} \cdot \frac{du}{-2 \cos x} = \frac{3}{2} \int u^{-1/2} du \\ &= 3 u^{1/2} + c = \boxed{3 \sqrt{4 - 2 \sin x} + c \quad c \in \mathbb{R}} \end{aligned}$$

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5.3 Linear substitution

E.g (1) $\int (x+2)\sqrt{3-x} \, dx = I$, say
 let $u = 3-x \Rightarrow \frac{du}{dx} = -1 \Rightarrow dx = -du$
 $x = 3-u$

This step is to name the original equation to easily refer back.

$$\begin{aligned} \therefore I &= \int (3-u+2) u^{1/2} (-du) \\ &= -\int (5-u) u^{1/2} du \\ &= \int (u-5) u^{1/2} du \\ &= \int (u^{3/2} - 5u^{1/2}) du \\ &= \frac{2}{5} u^{5/2} - 5 \cdot \frac{2}{3} u^{3/2} + c = \frac{2}{15} u^{3/2} (3u-25) + c = \frac{2}{5} (x-3)^{3/2} (16+3x) + c \\ &= \frac{2}{5} (3-x)^{5/2} - \frac{10}{3} (3-x)^{3/2} + c \quad (c \in \mathbb{R}) \end{aligned}$$

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