Table of contents for Specialist Maths

References to an activity, exercise or other type of question are from:
Coffey D, McLaverty J, Anastasio R, Matherson A. (2009). Specialist mathematics Enhanced. Australia: Pearson Heinemann.
What's found in here are our own solutions along with notes on specialist mathematics.

3. Coordinate geometry and graphs

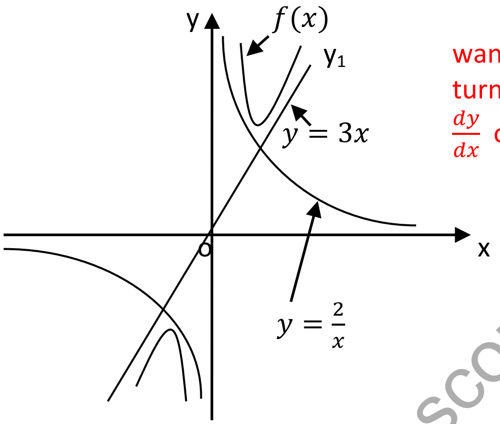
3.1 Addition of Ordinates

- $f(x) = ax + \frac{b}{x}$

$\Rightarrow f(x) = y_1 + y_2$

- sketch y_1 and y_2 separately then add them together to produce $f(x)$

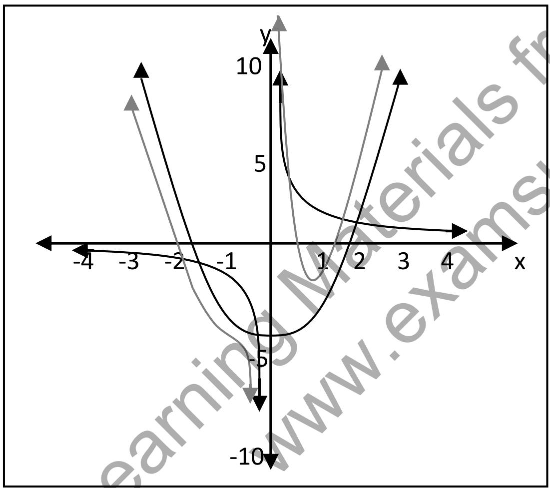
<u>Example 1</u>: $f(x) = 3x + \frac{2}{x}$



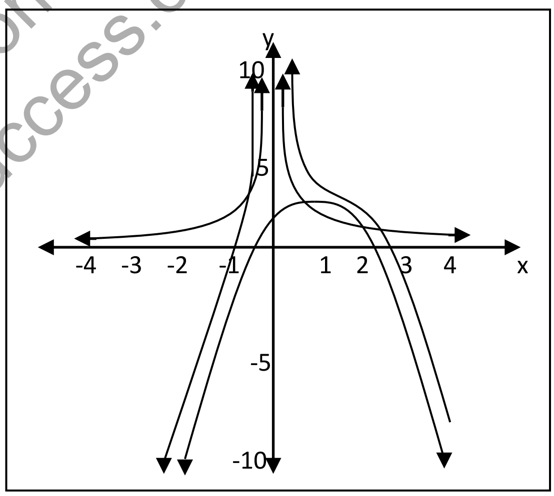
want to find the turning point, find $\frac{dy}{dx}$ of $f(x)$

<u>Example 2</u>: $y = \frac{x^2 - x + 2}{x} = \underbrace{x - 1}_{y_1} + \underbrace{\frac{2}{x}}_{y_2}$

$y = \frac{2}{x} + x^2 - 4$



$y = \frac{2}{x^2} - (x-1)^2 + 2$



3.2 Reciprocating Ordinates

1.**Reciprocating linear functions**

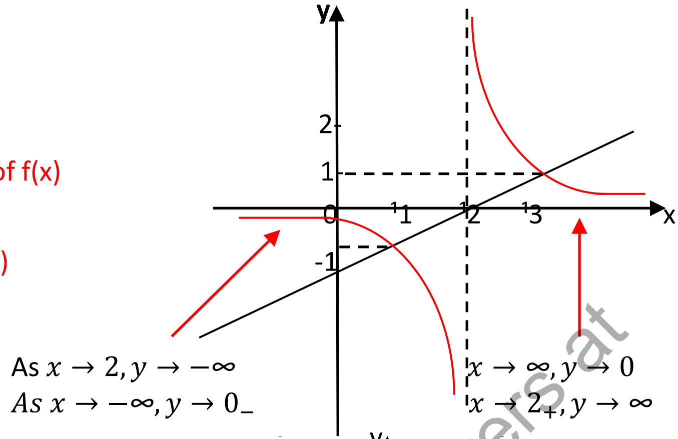
Example: $f(x) = x - 2$

$$y = \frac{1}{f(x)} = \frac{1}{x-2}$$

Notice: asymptotes at x-intercepts of f(x)

$$f(x) = \frac{1}{f(x)} \quad at \quad y = \pm 1$$

(points of intersection at $y = \pm 1$)



2.**Reciprocating quadratic functions**

Example 1: $f(x) = 4 - x^2$

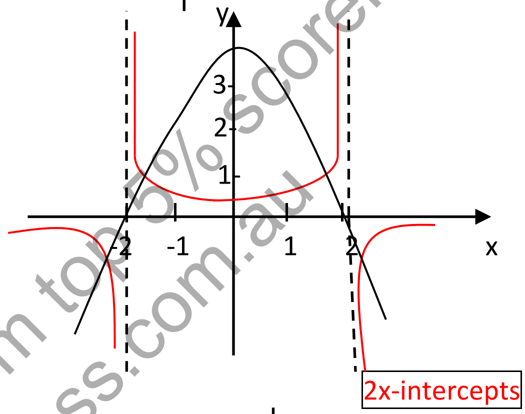
$$y = \frac{1}{f(x)} = \frac{1}{4-x^2}$$

* Even function has Oy- as the symmetrical axis.

$\Rightarrow g(-x) = g(x)$

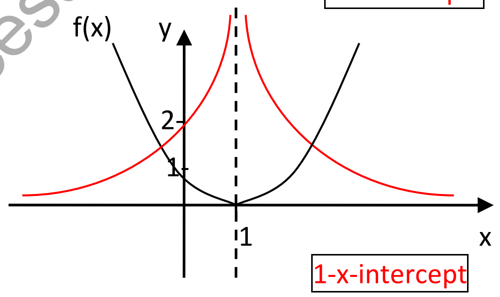
* Odd function such as $y = x,$

$y = x^3 \Rightarrow g(-x) = -g(x)$



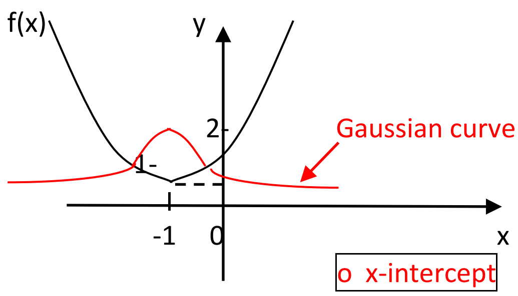
Example 2: $f(x) = \frac{1}{2}(x-1)^2$

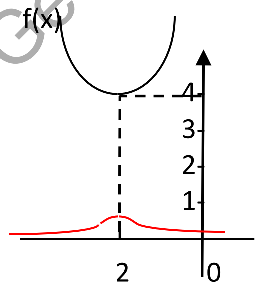
$$y = \frac{1}{f(x)} = \frac{2}{(x-1)^2}$$



Example 3: $f(x) = (x+1)^2 + \frac{1}{2}$

$$y = \frac{1}{f(x)} = \frac{1}{(x+1)^2 + \frac{1}{2}}$$





Example (4) $\int x\,(3x^2-11)^4\,dx$

$\Rightarrow \frac{1}{6}\int 6x(3x^2-11)^4\,dx = \frac{1}{6\times 5}(3x^2-11)^5 + c = \boxed{\frac{1}{30}(3x^2-11)^5 + c \quad c \in IR}$

Example (5) $\int \underbrace{(15x^2-7)}\,(5x^3-7x)^7\,dx = \boxed{\frac{1}{8}(5x^3-7x)^8 + \text{c} \quad \text{c} \in \text{IR}}$

exactly the differentiate inside

Example (6) $\int \cos x\,.\,e^{7\sin x}\,dx = \frac{1}{7}\int 7\cos x\,.\,e^{7\sin x}\,dx$

$= \boxed{\frac{1}{7}\,e^{7\sin x} + c \quad (c \in IR)}$

* $\int \frac{1}{x}\,dx = \log_e|x| + c, \;\; c \in IR$

(top is the derivative of bottom)

Example (7) $\int \frac{2}{2x-5}\,dx = \boxed{\log_e|2x-5| + c, \;\; c \in IR}$

Example (8) $\int \frac{x-2}{x^2-4x}\,dx = \frac{1}{2}\int \frac{2(x-2)}{x^2-4x}\,dx = \boxed{\frac{1}{2}\log_e|x^2-4x| + c \quad (c \in IR)}$

Important notice:

- * The expression $\int f(x)\,dx$ always needs "+c" in its answer.
- * If the question ask to find " an antiderivative" then please have an extra step at the end to state the final answer without "+c".

Look at example 4 if not clear.

5.2 Antidifferentiation using change of variable

Substitution method

Example 1. $\int 5x(x^2+3)^6\,dx = I,\ say$

let $u = x^2+3 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$\Rightarrow I = \int 5x\,.u^6.\frac{du}{2x} = \int \frac{5}{2}u^6\,du = \frac{5}{2}\int u^6\,du$

$= \frac{5}{2}\times\frac{1}{7}u^7 + c = \frac{5}{14}u^7 + c = \boxed{\frac{5}{14}(x^2+3)^7 + c \quad c\in IR}$

Example 2. $\int 3x\sqrt{x^2-1}\,dx = I,\ say$

let $u = x^2-1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$\therefore I = \int 3x\,.u^{1/2}.\frac{du}{2x} = \frac{3}{2}\int u^{1/2}\,du$

$= \frac{3}{2}\times\frac{2}{3}u^{3/2} + c = u^{3/2} + c = \boxed{(x^2-1)^{3/2} + c\ ,\ \ (c\in IR)}$

The answer should reflect the style of the question

$\therefore$ The final answer $= (x^2-1)\sqrt{x^2-1} + c$

Example 3. $\int (3x-2)\sqrt[3]{3x^2-4x}\,dx = I,\ say$

let $u = 3x^2-4x \Rightarrow \frac{du}{dx} = 6x-4 \Rightarrow dx = \frac{1}{6x-4}\times du$

$\therefore I = \int (3x-2).u^{1/3}.\frac{1}{6x-4}\,du$

$= \frac{1}{2}\int u^{1/3}\,du = \frac{1}{2}\times\frac{3}{4}u^{4/3} + c = \frac{3}{8}u^{4/3} + c \quad (c\in IR)$

$= \boxed{\frac{3}{8}(3x^2-4x)\sqrt[3]{3x^2-4x} + c}$

Example 4. $\int \frac{5x}{\sqrt{7x^2+3}}\,dx = I,\ say$

let $u = 7x^2+3 \Rightarrow \frac{du}{dx} = 14x \Rightarrow dx = \frac{1}{14x}\,du$

$$\therefore I = \int \frac{5x}{\sqrt{u}} \cdot \frac{1}{14x}\, du = \frac{5}{14} \int u^{-1/2}\, du$$

$$= \frac{5}{14} \times 2\, u^{1/2} + c = \boxed{\frac{5}{7} \cdot \sqrt{7x^2+3} + c \qquad c \in IR}$$

<u>Example 5</u> $\int 3\cos x . \sin^2 x\, dx = I,\ say$

let $u = \sin x \Rightarrow \frac{du}{dx} = \cos x \Rightarrow dx = \frac{1}{\cos x}\, du$

$$\therefore I = \int 3\cos x \,.\, u^5 \,.\, \frac{1}{\cos x}\, du = 3\int u^5\, du$$

$$= \frac{3}{6}\, u^6 + c = \boxed{\frac{1}{2}\sin^6 x + c \qquad (c \in IR)}$$

<u>Example 6</u> $\int 5\sin x\ e^{3\cos x}\, dx = I,\ say$

let $u = 3\cos x \Rightarrow \frac{du}{dx} = -3\sin x \Rightarrow dx = \frac{-1}{3\sin x}\, du$

$$\therefore I = \int 5\sin x \,.\, e^u \,.\, \frac{-1}{3\sin x}\, du$$

$$= \frac{-5}{3} \int e^u\, du = \frac{-5}{3}\, e^u + c$$

$$= \frac{-5}{3}\, e^{3\cos x} + c\ , \quad c \in IR$$

<u>Example 7</u> $\int \frac{-3\cos x}{\sqrt{4-2\sin x}}\, dx = I,\ say$

let $u = 4 - 2\sin x \Rightarrow \frac{du}{dx} = -2\cos x \Rightarrow dx = \frac{du}{-2\cos x}$

$$\therefore I = \int -3\cos x \,.\, \frac{1}{u^{1/2}} \,.\, \frac{du}{-2\cos x} = \frac{3}{2}\int u^{-1/2}\, du$$

$$= 3\, u^{1/2} + c = \boxed{3\sqrt{4-2\sin x} + c \qquad c \in IR}$$

5.3 Linear substitution

E.g (1) $\int (x+2)\sqrt{3-x}\,dx = I,\ say$

This step is to name the original equation to easily refer back.

let $u = 3 - x \quad \Rightarrow \quad \frac{du}{dx} = -1 \quad \Rightarrow \quad dx = -du$

$x = 3 - u$

$$\therefore I = \int (3 - u + 2)\ u^{1/2}\,(-du)$$

$$= -\int (5-u)\ u^{1/2}\,du$$

$$= \int (u-5)\ u^{1/2}\,du$$

$$= \int (u^{3/2} - 5\ u^{1/2})\,du$$

$$= \frac{2}{5}u^{5/2} - 5.\frac{2}{3}u^{3/2} + c = \frac{2}{15}u^{3/2}(3u-25) + c = \boxed{\frac{2}{5}(x-3)^{3/2}(16+3x) + c}$$

$$= \frac{2}{5}(3-x)^{5/2} - \frac{10}{3}(3-x)^{3/2} + c \qquad (c \in IR)$$